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A FEW GENEREALISED FORM OF TOPOLOGICAL SPACES OF THE SETS OF GRAPHS

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ABSTRACT

We know the concepts of open sets, closed sets, door- space, closure and dense sets of a topological space defined on any given set of elements. Here in this chapter we shall introduce all these concepts on the graphs. First we shall define all the definitions w.r.t. the topological space of any set of elements.

Key words: Topological space, element

INTRODUCTION

Open Set: Given topological space (X, T) any set A $\in$ T is said to be an open subset of X of T-open set.

Closed Set: Given a topological space (X,T), the complement in X of an open set belongs to is defined to be a closed set or T-closed set.

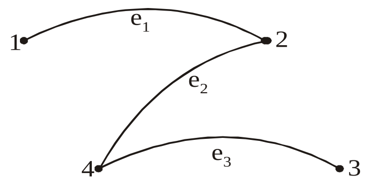
OR

Given a topological space (X, T) a subset a of X *i.e.* A $\subset$ X is said to be T closed set iff X ~ A $\in|$ T i.e. iff X ~ A is T- open.

Open Set: Given a topological space (E, T) defined on any set of edges, let E' T is said to an open set of E of T= open set

Example: let E {e_1, e_2, e_3] is an edge set on a simple graph G.

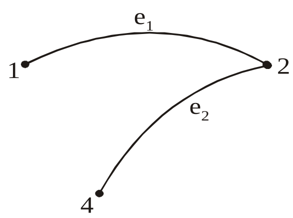
let $\exists|$ a topology T = {ϕ, {e_1. e_2} E}



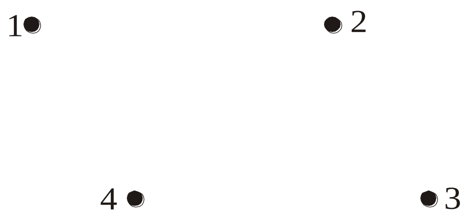
on the edge of = {e_1. e_2, e_3}

let E' $\in|$ T i.e. {e_1. e_2, } $\in|$ T *i.e.*

As E' $\in$ T, hence it called T- open set. Similarly

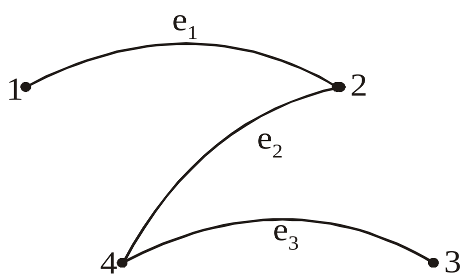


ϕ $\in$T i.e.



$\in$T

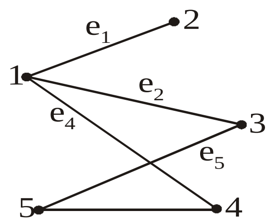
and E T *i.e.*



Thus ϕ & E are also T-open set.

Let us consider one more example for the better understanding of open sets with respect of edge set of graphs.

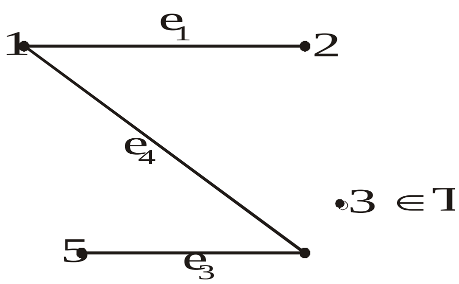
Example: Now taking an edge set of simple graph defined on the 5 edges.



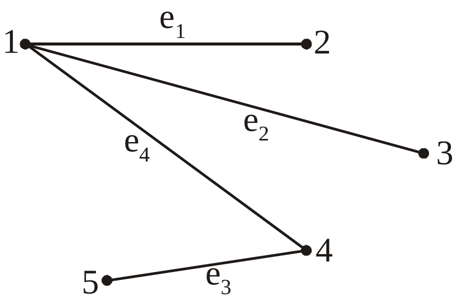
let E = {e_1, e_2, e_3, e_4, e_5}

let us a topology T = {ϕ {e_1,}, {e_1, e_2,} {e_1, e_3, e_4}, {e_1, e_2, e_3, e_4} E} on the edge set E.
As E' = {e_1, e_2, e_4} T hence E' is open subset of E or T-open set.

i.e.



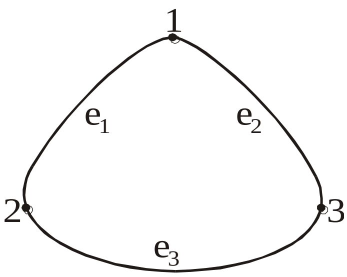
Similarly E" = {e_1, e_2, e_3, e_4) is also an open subset of E.
i.e.



∈ T

Hence all the sets belong to the topology T is said to be open subset of e. Similarly we can define the T-closed sets respect to edge set of simple graph.

Closed Set: Given a topological space (E, T), a subset E_1 of *i.e.* E_1 $E_1 \in E$ is said to be T-closed set if the complement E_1 i.e. E_1 a T-open set on E.
Example: consider an example set us take an edge set E= {e_1, e_2, e_3) and a topological space T = =(ϕ, E)

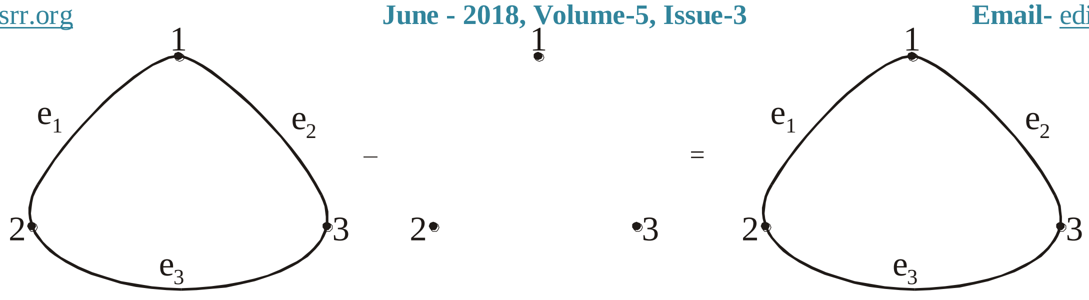


E

Since
E - ϕ = E and E = E = ϕ , thus E and ϕ are T-closed set. As these are the compliments of T-open set ϕ and E.

We can examine it with the help of the graph :

E = ϕ =

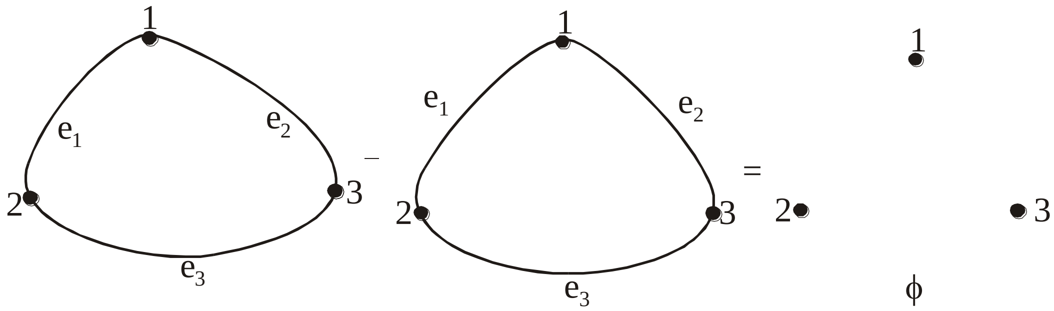


E

As E is complement of ϕ which belongs E to the topology, T, so E is T-closed.

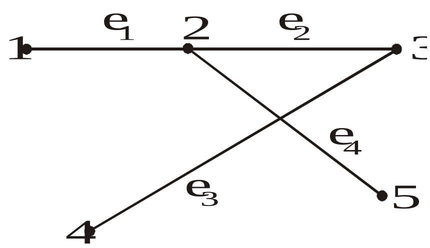
Similarly

E –E =

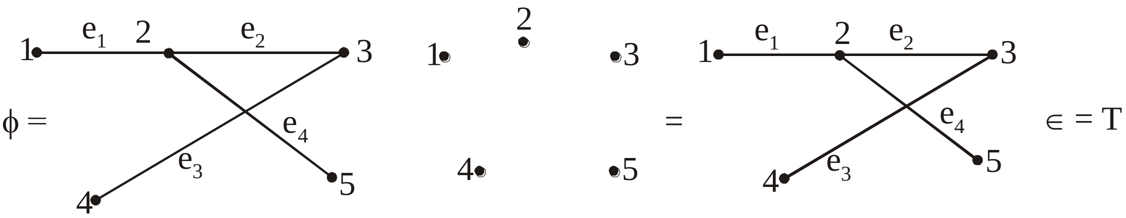


Similarly ϕ ∈ T, hence E is a T- closed set.
Similarly we can explain the same concept with the help of another example:
Example: let there exist a Topology T
= {ϕ {e_2}, { e_2, e_3} {e_2, e_4 }, { e_2, e_3, e_4,}, E} on the edge set.

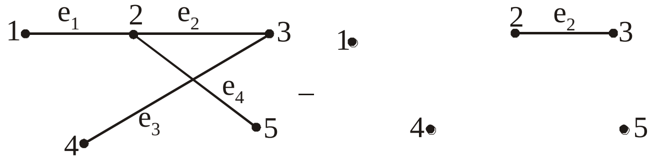
E = { e_1, e_2, e_3, e_4}. *i.e.*



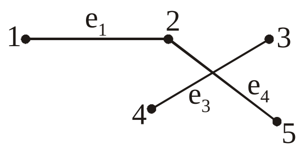
E – ϕ =



As the complement of i.e. E is an open set. Thus is a T-closed set
Similarly



=

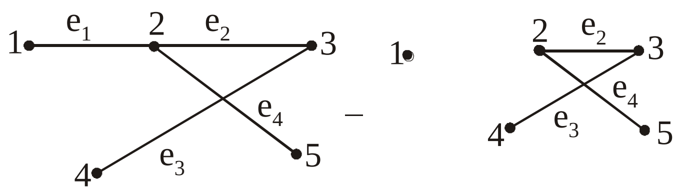


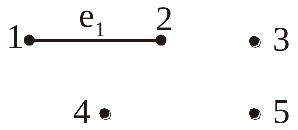
i.e. {e_1, e_3, e_4}

Thus $\{e_2\}$ and $\{e_1,e_3, e_4\}$ are complementary to each other, hence the complement of $\{ e_1,e_3, e_4\}$ E is $\{ e_2,\}$ which is a T- open hence $\{ e_1,e_2, e_4 \}$ is a closed set.
Same way

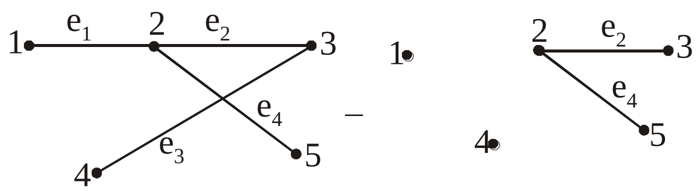
E $\{e_2, e_3, e_4\}$ =

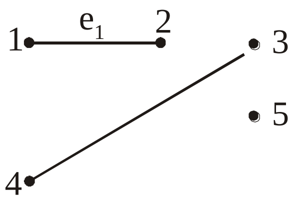
E = $\{e_2, e_4\}$ =





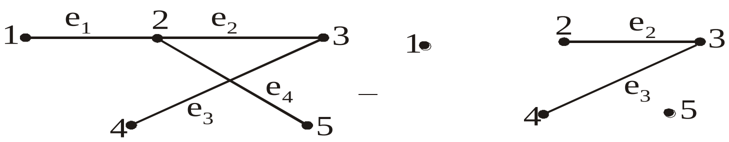
i.e. $\{e_1\}$

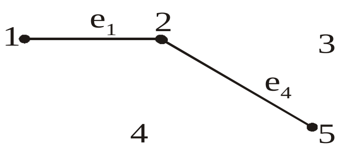




i.e. $\{e_1, e_3\}$

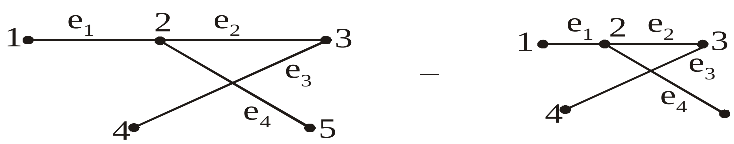
E – $\{e_2, e_3\}$ =

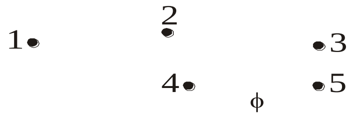




i.e. $\{e_1, e_4\}$

and E – E =





Thus E, $\{e_2. e_3\}$, $\{e_2. e_4\}$, $\{e_1. e_3\ e_4\}$ are the complement of $\{ e_1, e_4\}$ $\{e_1, e_3\}$ and $\{ e_1 \}$. Thus $\{e_1. e_4\}$, $\{e_1. e_3\}$, & $\{e_1\}$ are also T- closed sets.

Theorem 1: If (E,T) be a topological, then

(i) any arbitrary intersection of closed set in E is closed..

(ii) any finite union of closed sets is E is closed.

Proof: let the edge set is E= $\{e_1, e_2, e_3)$ and the topology

$T = \{\phi\ E, \{e_1\}, \{e_1, e_2\}, \{e_1. e_3\}\}$.

(i) let $\{E_a : \alpha \in \wedge\}$ be an arbitrary of collection of closed sets of E.

Now to show $\{E_\alpha : \alpha \in \wedge \}$ is a T closed set.

let $E\alpha$ is T-closed, $\forall\ \alpha \in \wedge$

$\Rightarrow \cup - E\alpha\ \ E$ is open, $-\forall\ \alpha \in \wedge$

$\Rightarrow\ \cap\ \{ \cup -E\alpha : \alpha \in \wedge \}$ is open by the (ii) axiom of topology.

$\Rightarrow\ \cup\ \{\cap\ -\{E\alpha : \alpha \in \wedge \}$ is T- open by Demorgan's law.

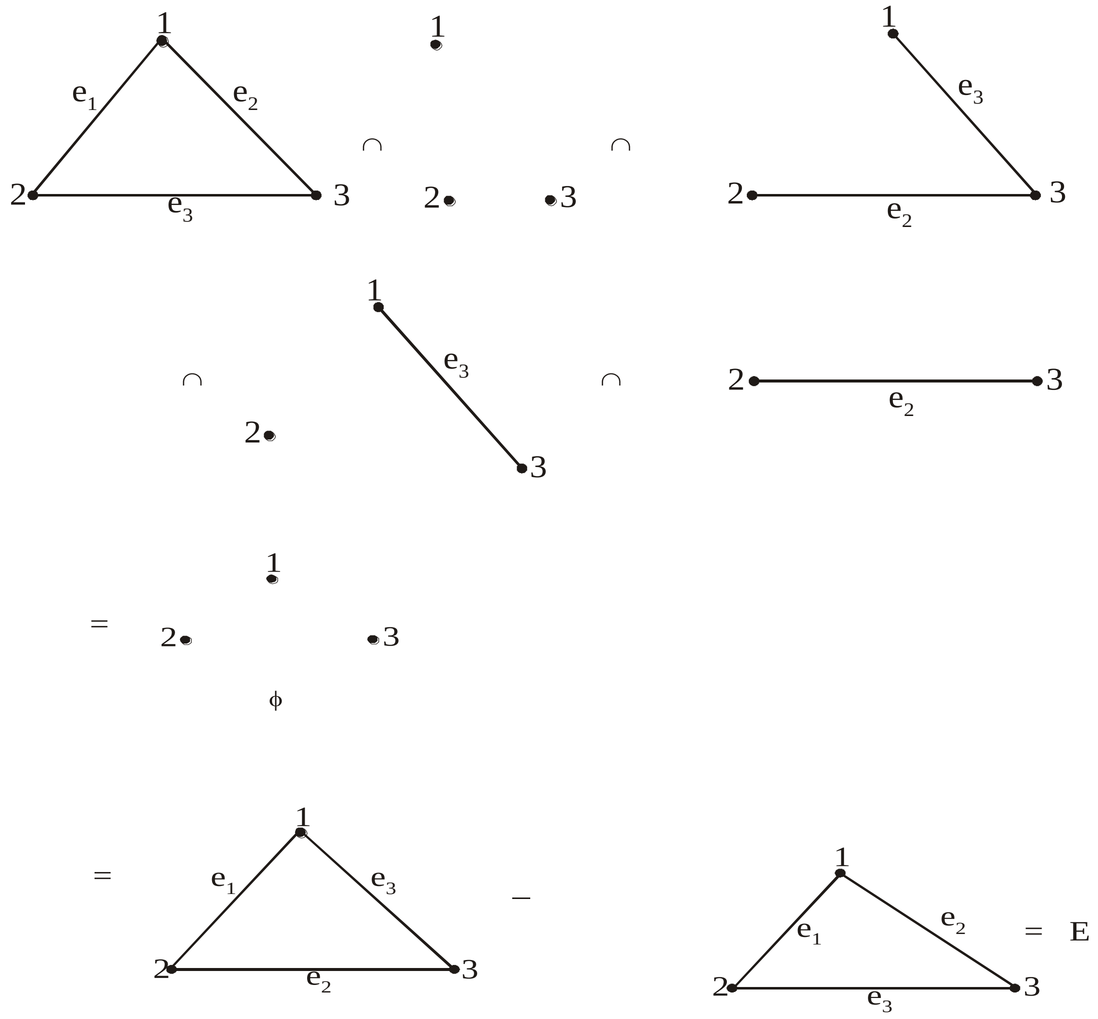
$\Rightarrow\ \cap\ \{E\alpha : \alpha \in \wedge \}$ is T- closed, as the complement of an open set is a closed set.

Now we will verify it with the above given topologies.

the T-closed set are E, ϕ $\{ e_2, e_3,\}$ $\{e_3\}$, $\{e_2\}$.

let $\cap\ E\alpha$, where are $E\alpha$'s closed sets.

i.e. $E \cap \phi \cap| \{e_2, e_3) \cap \{e_3) \cap \{e_2)$



Hence $\cap\ E\alpha$, $= \phi = E'$ complement of an open set E.

Thus intersection of finite family of closed set is closed.

(ii) let E_1 & $E_2 \lhd$ E, be T- closed. Then $E_1, E_2 \subset E$ are T- closed.

$\Rightarrow$ E-E_1 & E-E_2 are T open

$\Rightarrow$ (E-E_1) $\cup$ (E-E_2) are T- open by the axiom of topology T_3.

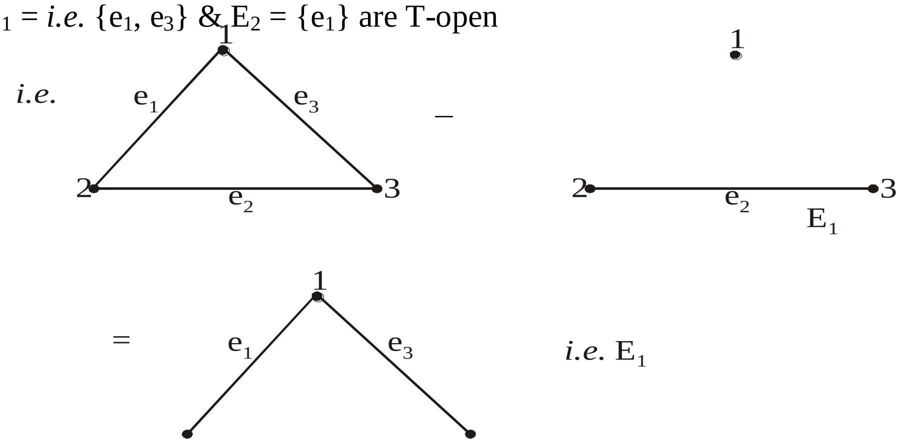
$\Rightarrow$ E- ($E_1 \cup E_2$) is T open by demorgan law

$\Rightarrow$ ($E_1 \cup E_2$) is T-closed, as the complement of open set is closed.

Now let us verify it with the help of an example :

let $E_1 = \{e_2\}$, $E_2 = \{e_2, e_3)$ are closed sets.

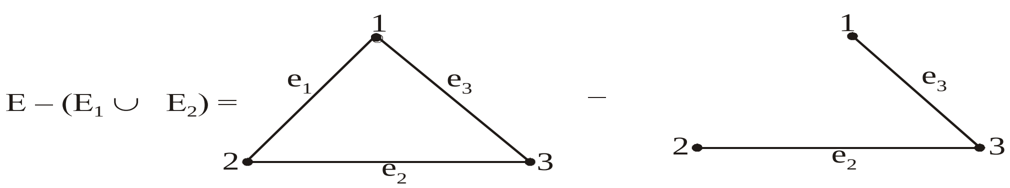
$\Rightarrow$ E_1 = *i.e.* $\{e_1, e_3\}$ & $E_2 = \{e_1\}$ are T-open

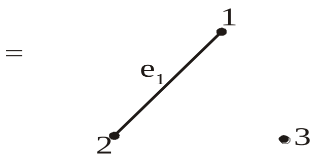


hus by $\Im_3$ it is in T.

By demorgan E- ($E_1 \cup E_2$) is T- open

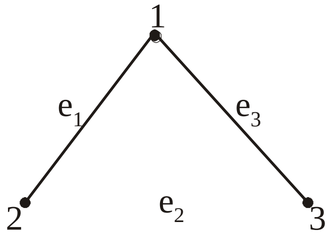
i.e. E- $\{E_1\ E_2\} \in$ T

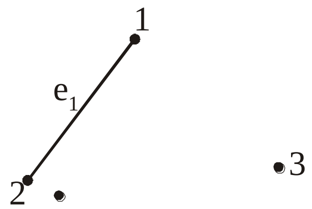


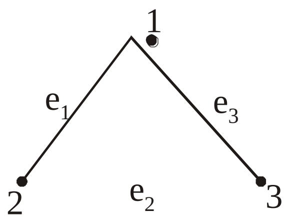


$\in$ T

$\Rightarrow$ $E'_1 \cup E'_2$ is also T – open

$E'_1 \cup E'_2 =$ 

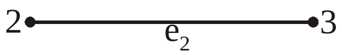
$\cup$ 

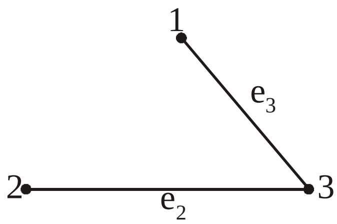
$=$ 

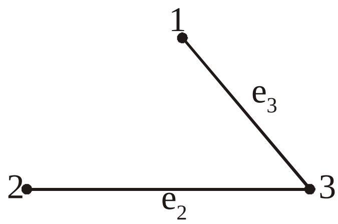
$\in$ T

Hence E- ($E_1 \cup E_2$) is T- open.

$\Rightarrow$As the complement is open, thus by def. of closed set, set is closed. Hence ($E_1 \cup E_2$) is closed.



$\cup$ 

$=$ 

$\Rightarrow$Closed

Now, we will study another concept Door Shape with respect to the edge set of the simple graph.

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